3.4 Elimination, Part 2

In Part 1 of lesson 3.4, we always had opposite coefficients, and could therefore skip step 2. What happens when we don't have opposite coefficients?

Example 3. $\begin{cases} -x + 3y = -7 \\ 9x + 3y = -27 \end{cases}$

In this case, we don't have any opposite coefficients. However, we can still eliminate a variable.

Step 1: Let's try to eliminate y. Currently, we have $3y$ and $3y$ – if we want coefficients, we'd really like $3y$ and $-3y$.

Step 2: To turn $3y$ into $-3y$, we need to multiply by -1. Pick one of the equations (it really doesn't matter which in this case) and multiply all of the terms by -1. This time, I'll pick the first equation.

$$-x + 3y = -7$$

$$-1(-x) + (-1)(3y) = (-1)(-7)$$

$$x - 3y = 7$$

Step 3: Add the equations. We're going to add our equation from step 2 and the equation we did not change. If we've done everything correctly up to this point, y should disappear.

$$\begin{array}{r} x - 3y = 7 \\ +\ 9x + 3y = -27 \\ \hline 10\,x + 0 = -20 \end{array}$$

Step 4: Solve the new equation.

$$\frac{10x}{10} = \frac{-20}{10}$$

$$x = -2$$

Step 5: Pick one of the original equations (I'm going with the first equation again, here), substitute in -2, and solve for y.

$$-x + 3y = -7$$

$$-(-2) + 3y = -7$$

$$\begin{array}{r} 2 + 3y = -7 \\ -2 \qquad\quad -2 \end{array}$$

$$\frac{3y}{3} = \frac{-9}{3}$$

$$y = -3$$

Therefore our final solution is $(-2, -3)$.

This next example will be solved twice – not because you need to solve it twice, but because there are a couple of different ways to solve it.

Example 4. $\begin{cases} 5x + y = -6 \\ -15x + 3y = 12 \end{cases}$

First option:

Step 1: Let's eliminate y. Coefficients are currently 1 and 3; we want -3 and 3.

Step 2: Multiply the first equation by -3 to create opposite coefficients on y.

$$5x + y = -6$$

$$-3(5x) + (-3)(y) = (-3)(-6)$$

$$-15x - 3y = 18$$

Step 3: Add the equations.

$$\begin{array}{r} -15x - 3y = 18 \\ + \quad -15x + 3y = 12 \\ \hline -30x + 0 = 30 \end{array}$$

Step 4: Solve.

$$\frac{-30x}{-30} = \frac{30}{-30}$$

$$x = -1$$

Step 5: Substitute and solve. I'm going to choose the first equation.

$$5x + y = -6$$

$$5(-1) + y = -6$$

$$\begin{array}{r} -5 + y = -6 \\ +5 \qquad\; +5 \end{array}$$

$$y = -1$$

Therefore the solution is $(-1, -1)$.

Second option:

Step 1: Let's eliminate x. Coefficients are currently 5 and -15; we want 15 and -15.

Step 2: Multiply the first equation by 3 to create opposite coefficients.

$$5x + y = -6$$

$$3(5x) + 3(y) = 3(-6)$$

$$15x + 3y = -18$$

Step 3: Add the equations. x should disappear.

$$\begin{array}{r} 15x + 3y = -18 \\ + \quad -15x + 3y = 12 \\ \hline 0 + 6y = -6 \end{array}$$

Step 4: Solve.

$$\frac{6y}{6} = \frac{-6}{6}$$

$$y = -1$$

Step 5: Substitute and solve. I'm going to choose the first equation.

$$5x + y = -6$$

$$\begin{array}{r} 5x + (-1) = -6 \\ +1 \quad\; +1 \end{array}$$

$$\frac{5x}{5} = \frac{-5}{5}$$

$$x = -1$$

Therefore, the solution is $(-1, -1)$.

The same solution can be found whether we choose to eliminate x or eliminate y. Often, it will be easier to eliminate one over the other; however, deciding which route will be easier can only be gained through experience. Both routes are equally mathematically valid.

In all my previous examples, we were multiplying the first equation. It is perfectly valid math to multiply the second equation. For example, in the system $\begin{cases} -10x + 4y = 28 \\ \ \ 5x + 2y = 10 \end{cases}$ you would multiply the second equation by 2 to eliminate x, or -2 to eliminate y.

Another option is to *divide* all of the terms in one equation by a number. For example, in $\begin{cases} -10x + 4y = 28 \\ \ \ 5x + 2y = 10 \end{cases}$, you could divide all of the terms in the first equation by 2, which would allow you to eliminate x, or by -2 to eliminate y. This is not the usual route, however, because there is a greater probability that we would find ourselves working with fractions. For instance, in the system $\begin{cases} 2x - 3y = 8 \\ -x + 4y = 6 \end{cases}$, it is mathematically valid to divide the first equation by 2. However, you would get the result of $x - \frac{3}{2}y = 4$; mathematically valid, but not fun to work with.

What about multiplying both equations? Let's look at the example $\begin{cases} 3x - 7y = -6 \\ 4x + 2y = -8 \end{cases}$

Step 1: Let's get rid of y.[1] Currently, the coefficients are -7 and 2. The LCM[2] is 14, so I want the coefficients on y to be -14 and 14.

Step 2: This step is a little more complicated this time, because I have to multiply *both* equations to get them to cancel. I'll need to multiply the first equation by 2 to turn -7 into -14.

$$3x - 7y = -6$$

$$2(3x) + 2(-7y) = 2(-6)$$

$$6x - 14y = -12$$

I'll also have to multiply the second equation by 7 to turn 2 into 14.

$$4x + 2y = -8$$

$$7(4x) + 7(2y) = 7(-8)$$

$$28x + 14y = -56$$

This means I now have the system $\begin{matrix} 6x - 14y = -12 \\ 28x + 14y = -56 \end{matrix}$ with the same solutions as the first system, and I can now add and solve as before.

Let's look at one last example, this one involving a word problem.

[1] It's perfectly acceptable to get rid of x. I just decided I liked y

[2] Least Common Multiple. When looking for your opposite coefficients, you should always use the LCM.

> Jennifer and Peter are selling wrapping paper for a school fundraiser. Customers can buy rolls of plain wrapping paper and rolls of holiday wrapping paper. Jennifer sold 7 rolls of plain wrapping paper and 4 rolls of holiday wrapping paper for a total of $85. Peter sold 1 roll of plain wrapping paper and 4 rolls of holiday wrapping paper for a total of $43. Find the cost each of one roll of plain wrapping paper and one roll of holiday wrapping paper.

Let's start by defining some variables. We know that we're looking for the cost of the rolls of paper.

Let p be the price of one roll of plain wrapping paper (in dollars).

Let h be the price of one roll of holiday wrapping paper (in dollars).

From here, we can start creating equations. In this case, we have two scenarios: how much Jennifer sold, and how much Peter sold. Let's start by creating an equation for Jennifer.

"Jennifer sold 7 rolls of plain wrapping paper and 4 rolls of holiday wrapping paper for a total of $85."

The price of 7 rolls of plain wrapping paper is $7p$, and the price of 4 rolls of holiday is $4h$. These two added together gives us our total. That makes Jennifer's equation $7p + 4h = 85$.

Peter's equation is similar, except that he only sold 1 roll of plain paper and made $43. His equation is $1p + 4h = 43$.

This gives us the system $\begin{matrix} 7p + 4h = 85 \\ 1p + 4h = 43 \end{matrix}$, which we can solve using the elimination method we've been practicing!

Let's get rid of h. That means we'll have to multiply the second equation by -1 to have opposite coefficients.[3][4]

$$1p + 4h = 43$$

$$-1(1p) + -(4h) = -1(43)$$

$$-1p - 4h = -43$$

Add the equations:

$$\begin{array}{r} 7p + 4h = 85 \\ + \quad -1p - 4h = -43 \\ \hline 6p + 0 = 42 \end{array}$$

$$\frac{6p}{6} = \frac{42}{6}$$

$$p = 7$$

We now know that the price of one roll of plain paper is $7. Let's use that information to find h.

---

[3] You don't *have* to write it as $1p$; you could just write p.

[4] You could multiply the first equation. When multiplying by -1, I tend to choose the equation with smaller numbers, in the hope that I'll have fewer negatives to keep track of later!

$$p + 4h = 43$$

$$\begin{aligned} (7) + 4h &= \quad 43 \\ -7 \qquad & \quad -7 \end{aligned}$$

$$\frac{4h}{4} = \frac{36}{4}$$

$$h = 9$$

Therefore,

One roll of plain wrapping paper costs $7, and one roll of holiday wrapping paper costs $9.