3.4 Solving by Elimination

When both of the equations in the system are in slope-intercept form, solving by graphing can work really well. But when both equations are in standard form, graphing becomes more difficult. Another method to solving systems of equations is the *elimination method*, which has the goal of creating a new equation with only one variable.

Steps to the elimination method:

1. Determine which variable should be eliminated.
   a. Look at the coefficients. *Opposite coefficients* (same number, opposite sign) indicate the easiest variables to eliminate.
2. If necessary, create opposite coefficients by multiplying one or both equations in the system.
3. Add the equation together.
   a. If the first few steps have been executed properly, one of the variables will disappear.
4. Solve the resulting equation.
   a. You now have half of the final solution.
5. Choose one of the original equations, substitute the value from step 4 for the proper variable, and solve.

Example: $\begin{cases} x - 3y = 11 \\ -x + 4y = -16 \end{cases}$

Step 1: The coefficients of x are 1 and -1, which are opposites. This means that x is a good choice to try and eliminate.

Step 2: We already have opposite coefficients. Skip step 2!

Step 3: Add the equations.

$$\begin{array}{r} x - 3y = 11 \\ + \quad -x + 4y = -16 \\ \hline 0x + 1y = -5 \end{array}$$

$0x$ comes out to 0, so x has disappeared from the new equation, which is what we were hoping for.

Step 4: Now we need to solve the new equation. In this case, that is very simple:

$$1y = -5$$

$$y = -5$$

Step 5: Because there were two variables in our original equations, we need two numbers in our final solution, and we only have 1. We need to substitute the value we got back into one of the original equations to finding the matching value.

Because a point is only a solution if it makes *all* of the equations in the system true, it doesn't matter which equation we pick to work with; we'll get the same answer. Personally, I like to pick the equation with the smaller or more positive coefficients and constants.

$$x - 3y = 11$$

$$x - 3(-5) = 11$$

$$x + 15 = 11$$

$$\begin{aligned} x + 15 &= 11 \\ -15 \quad &\; -15 \end{aligned}$$

$$x = -4$$

Because $y = -5$ and $x = -4$, our final solution is $(-4, -5)$.

How about another example?

Example 2. $\begin{cases} -5x + 4y = 23 \\ -2x - 4y = -2 \end{cases}$

Step 1: In this case, the coefficients of x are -5 and -2: not even close to being opposites! But we have better luck with the coefficients of y: 4 and -4. Let's eliminate y, this time.

Step 2: We already have opposite coefficients. On to step 3!

Step 3: Add the equations.

$$\begin{array}{rr} & -5x + 4y = 23 \\ + & -2x - 4y = -2 \\ \hline & -7x + 0y = 21 \end{array}$$

$$-7x = 21$$

Now y has disappeared, which is what we wanted.

Step 4: Divide by -7 to isolate x and solve the new equation.

$$\frac{-7x}{-7} = \frac{21}{-7}$$

$$x = -3$$

Step 5: Pick an equation, substitute -3 for x, and solve.

$$-2x - 4y = -2$$

$$-2(-3) - 4y = -2$$

$$\begin{aligned} 6 - 4y &= -2 \\ -6 \quad &\; -6 \end{aligned}$$

$$\frac{-4y}{-4} = \frac{-8}{-4}$$

$$y = 2$$

Therefore, our final solution is $(-3, 2)$.

Note: It is possible for both variables to disappear! If the resulting equation is true, the equations are the same line and there are infinite solutions. If the resulting equation is false, the equations are parallel lines and there are no solutions.

Ex. $\begin{aligned} -4x - 3y &= 9 \\ 4x + 3y &= -9 \end{aligned}$

$$0x + 0y = 0$$

$$0 = 0$$

Infinite Solutions